Check That Digit

NAME _____

How does a register at the supermarket accurately scan item numbers? Can mistakes in scanning be detected? The purpose of this activity is to see how modular arithmetic is applied to UPC and ISBN bar coding. Credit card numbers also use modular arithmetic to verify card numbers.

1. Verify the check digit for each of the two UPC numbers by doing the following steps:
   a) Every even-positioned digit, counting from the *right* to *left*, will be multiplied by 3. All odd-positioned digits will be multiplied by 1.
   b) Sum the products from step a.
   c) Determine the validity by dividing the sum by 10. If the remainder is 0, the UPC number is valid.

   a) 0-87684-00974-3

   b) 0-43197-11682-6

2. A problem of the UPC system is that if two adjacent digits that were transposed have a difference of 5, the error will not be detected. Explain why this occurs.

3. In general, larger publishing companies have a small identification number (the second block of the ISBN) and the smaller companies have a larger number. Explain why this is true.

4. For a publishing company that has 81 as its publisher identifier and 1 for its language/country identifier, determine the number of possible editions this publisher may print. The digits may be repeated. (Note: The ISBN number is 9 digits plus a check digit.)

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5. What if the digits were not repeated. How would this affect the number of published editions? What would be the total possible?

6. The ISBN system is better at detecting errors, specifically transposition errors. Question 2 addressed this problem with the UPC system. Explain why, unlike the UPC system, the ISBN system will detect all transposition errors.

7. As we have seen on many television commercials, there are a lot of banking institutions that offer credit cards. The first six digits that appear on a credit card are used for the issue identifier. How many possible issuers are there given each digit 0-9 could be used more than once?

8. Given that a credit number can have as many as 19 digits, six of which are reserved for the issue identifier and the last is the check digit, determine the total number of credit card numbers that are available to each issuer.

9. MasterCard issues identifier numbers are 6 digits in length and begin with either 51 or 55. How many possible issue identifier numbers are there for MasterCard?

10. Using the Luhn algorithm, determine the check digit for an account number of
601143871005123__
The Luhn algorithm is as follows:
1. Begin by doubling all even-positioned digits when counting from right to left.
2. Determine the sum *of the digits* from the products (Step 1) and each of the unaffected (odd-positioned digits) digits in the original number.
3. Determine the number to be added to the sum from Step 2 so that this new sum is a multiple of 10. This number is the check digit.

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11. In the above process there are two sums you are finding: one involving the odd-positioned digits, and the other involving the digits of the even-positioned numbers that were doubled. First, consider the sum of the odd-positioned digits. What is the most that that sum could be affected if an incorrect digit were entered? Explain why this occurs.

12. Next consider the even-positioned digits that were doubled. It is possible here to have numbers that result in double digits. When that happens, the digits are added to the sum. For instance, if 6 is doubled to make 12, then $1 + 2 = 3$ is added to the Luhn algorithm sum. How is it possible that a single digit error would be detected? [Hint: Make a table that shows possible values for the check digit, the double of the check digit, and the resulting sums.]

13. The Luhn algorithm is also used to detect most digit transpositions. For instance, when entering a number *5832403*, the data entry error is transposing the second and third digits *5382403*. There are two digits, when transposed, that will go undetected using the Luhn algorithm. What are they? Explain why this error cannot be detected.

14. Are there any other circumstances under which a transposition error would go undetected? If so, would this cause a problem?

15. What other types of accounts or cards would you expect may use a process similar to the Luhn algorithm?